***2015 Excellence in Mathematics Contest***

***Team Project***





School Name:

Group Members:

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**Formulas and Facts**

You may need to use some of the following formulas and facts in working through this project. You may not need to use every formula or each fact.

Area of a rectangle Perimeter of a rectangle Area of a circle

**  **

Circumference of a circle Area of a triangleSlope

12 inches = 1 foot 5280 feet = 1 mile 3 feet = 1 yard

16 ounces = 1 pound 2.54 centimeters ≈ 1 inch 100¢ = $1

1 kilogram ≈ 2.2 pounds 1 ton = 2000 pounds 1 gigabyte = 1000 megabytes

1 mile = 1609 meters 1 gallon ≈ 3.8 liters 1 square mile = 640 acres

1 sq. yd. = 9 sq. ft 1 cu. ft. of water ≈ 7.48 gallons 1 ml = 1 cu. cm.

Volume of cylinder Volume of rectangular prism Volume of a sphere

Lateral SA =   

Lateral surface area of cylinder Quadratic Formula

**TEAM PROJECT**

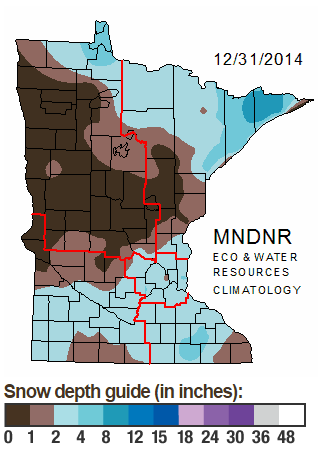
**2015 Excellence in Mathematics Contest**

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The Team Project is a group activity in which the students are presented an open ended, problem situation relating to a specific theme.  The team members are to solve the problems and write a narrative about the theme which answers all the mathematical questions posed.  Teams are graded on accuracy of mathematical content, clarity of explanations, and creativity in their narrative.

Part 1 – Introduction

Minnesota…the Land of 10,000 Lakes! With 76 state parks and recreation areas and more than 1,000 miles of park trails, winter scenery and activities like skiing, snowmobiling, sledding and snowshoeing are always within reach. Many state parks offer snowshoe and ski rentals and heated picnic shelters, and some feature winter lodging. Winter tourism in Minnesota can bring an economic boost to many areas of the state. So far this winter (2014-2015), however, there has not been much snow and Minnesotans are not able to do all the things that they love to do!

As of 12/31/2014, the following map shows snow accumulation through the state of Minnesota ([www.dnr.state.mn.us](http://www.dnr.state.mn.us)).

In this project, you will explore three aspects of Minnesota – particularly, Itasca County in Northern Minnesota.

First, you will explore, in spite of the current lack of snow, the building of a Minnesota snowman (who may or may not resemble Olaf). Next, you will explore something that many Minnesotans love – ice fishing! Particulalry, you will investigate the formation of ice on Minnesota’s 10,000 lakes. Finally, you will explore an industry that is very important to northern Minnesota – iron ore mining. Iron ore mining provides many jobs for Minnesotans as well as a natural resource used in the making of steel that benefits people around the world.

Have fun exploring, virtually, the beautiful state of Minnesota – Land of 10,000 Lakes!



Part 2 – The Snowman



When the snow conditions are right and the snow is sticky, Minnesotans enjoy spending time making a snowman. As shown, conditions were just right in December 2014. In fact, the snow was very “sticky” which means it was somewhat wet due to several consecutive days of temperatures in the mid-30’s (Fahrenheit), which is quite warm for that area in December! The snow was so heavy that it became a challenge to lift each of the nearly spherical parts of the snowman!

1. Use the following photos and measurements to determine the weight, in pounds, of the snowman. Clearly indicate any assumptions you need to make in order to determine the weight of the snowman.



250 grams

(same snowball shown below)



|  |  |
| --- | --- |
| Bottom Diameter | 26 inches |
| Middle Diameter | 19 inches |
| Head Diameter | 13 inches |
| Height | 61 inches |

Answers will vary. Students will likely assume that each section of the snowman is spherical and that the weight is proportional to the weight of the snowball shown. The work below shows a sample response.

Snowball: and weighs 250 grams

Bottom: and weighs 

Middle: and weighs 

Head: and weighs 

Total mass: 245,543.05 grams or 541.33 pounds.

Part 2 – Continues…

2. At some point, the snowman will melt (maybe not until May or June!). Suppose that all the water could be collected as the snowman melts. How many gallons of water would be collected? Again, clearly indicate any assumptions that need to be made in order to answer this question.

Using the computations from part one and continuing to assume that each section of the snowman is a sphere, the total volume: 13,944.48 cubic inches or

.

3. How much does one gallon of water weigh? Clearly explain how you know.

Based on the work done previously, we can conclude that one gallon of water weighs

.

Part 2 – Continues…

4. Suppose the snowman was made out of standard marshmallows instead of the wet snow of Minnesota. How much would he weigh? How many standard marshmallows would be needed to make this snowman? Clearly note any and all measurements you needed to answer these questions.

Students will have access to all necessary materials to make any necessary measurements. Answers will vary. Look for evidence of sound mathematical thinking and accurate conclusions. Here is a sample, brief, response:

A 10-ounce bag of Kraft Jet-Puffed Marshmallows has 40 marshmallows and so each marshmallow weighs 0.25 ounce.

If we take a single marshmallow to be roughly a right cylinder with a diameter of 1 inches and a height of 1inches, it has a volume of: .

It takes to make the snowman. Since each marshmallow weighs 0.25 ounce, the total weight of the marshmallow snowman is

or 117.14 pounds.

Part 3 – Frozen Lakes



Minnesota is known as the “Land of 10,000 Lakes” and these lakes provide recreation year round! During the cold winter months, the lakes freeze over and Minnesotans take advantage of the ice by skating, snowmobiling and even ice fishing!

The following description of how a lake freezes can be found at www.fishingvermont.net. It is included here just to provide background information that may be useful as you continue this part of the project.

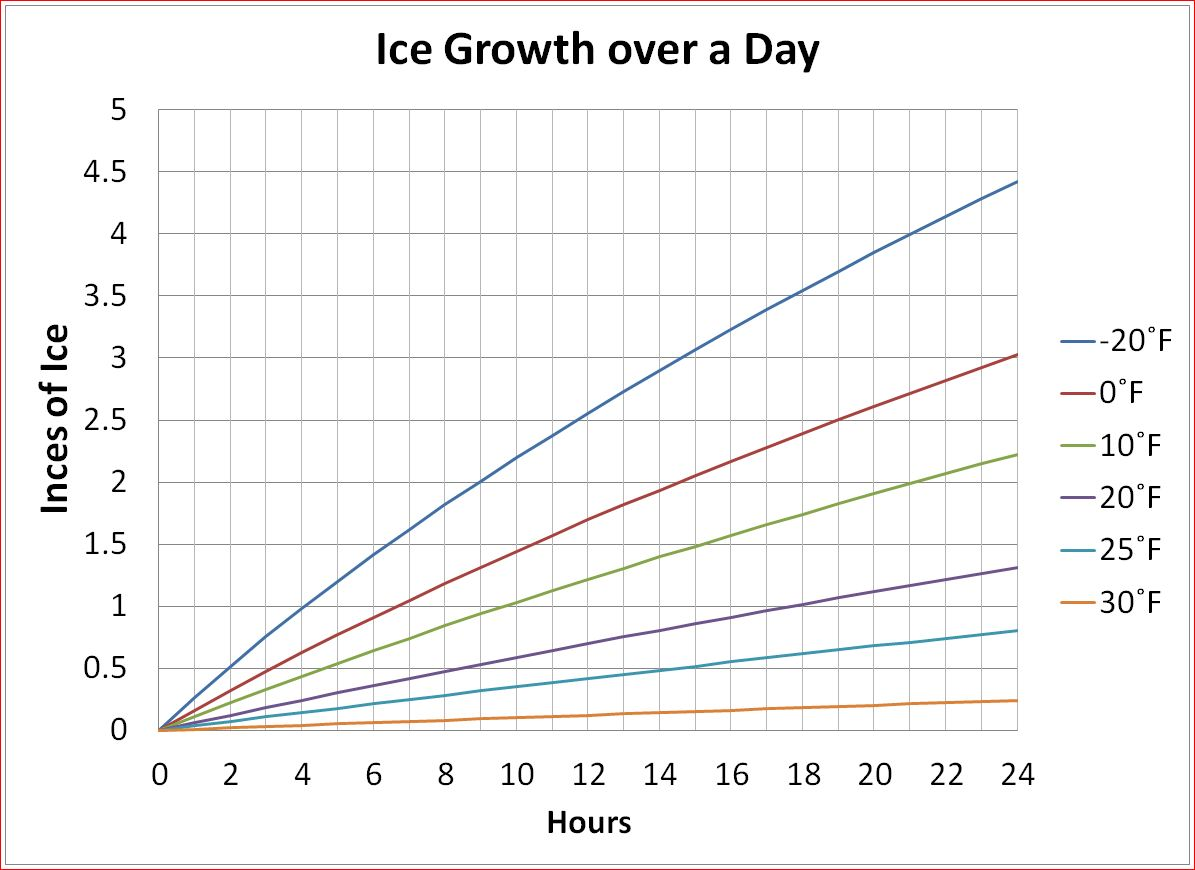
*For water to freeze and change into ice, it must be cooled to its freezing point, which implies heat loss. That heat loss occurs when the ambient temperature of the air is lower than the temperature of the water.  
The freezing point of fresh water is 0°C; however, its maximum density is reached at 4°C. From an ecological perspective, the latter point is extremely important, since the deeper water which is located under the ice – except in some small lakes or ponds - does not freeze, which means that the creatures living there can survive winter under the ice. Let us take the example of a lake whose temperature decreases to 4°C in the fall. Since density (the weight of the water per unit of volume) increases with the decrease in temperature, the entire lake – from the surface to its bottom – will reach 4°C after a certain length of time. When it cools, the thin layer close to the surface of the lake becomes denser, therefore heavier, and sinks toward the bottom of the lake, and is replaced on top by “warmer” water which rises to the surface.  
If the temperature continues to drop and dips below 4°C, the layer close to the surface becomes less dense when approaching the freezing point; it then increases in volume instead of shrinking, and becomes “lighter” than the water below it at 4°C. Therefore, that cooler layer will float on the surface and will continue to cool until ice is formed. Ice (a solid) is lighter than water (a liquid) due to its larger volume, and that is why it floats. The water located under the ice below remains at 4°C, except for the layer just below the ice’s surface. That layer will approach the freezing point of 0°C. As it reaches that temperature, it turns into ice as well, making the layer of ice on the surface even thicker. The colder it gets, the thicker and the more solid the layer of ice becomes.  
If water behaved like other physical bodies, it would increase in density as it cools. If that were true, rivers, streams and lakes would be frozen from the surface right down to the bottom. In the summer, only the top layer would melt, while the deeper water could remain frozen throughout the year. In such conditions, it is obvious that aquatic life in our rivers and lakes would be impossible.*

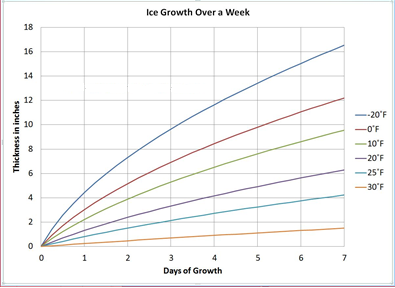
Once the first layer of ice forms on a lake it grows thicker at rate that is dependent on air temperature, wind speed and radiational cooling. Temperature is the easiest to assess.  Freezing degree days (FDD) are the average number of degrees below freezing over 24 hours. For example if the average temperature over a day is 17 degrees, that day had fifteen FDD’s.  An ice sheet will, in theory, grow at a rate of roughly one inch per fifteen FDDs starting from ice between 1/2" and 3" thick (as the ice gets thicker the growth rate decreases).  This is based on there being a bit of wind, a reasonably clear sky and no snow/frost on the ice.  If there is no wind or there are cloudy skies, the ice will grow about half as fast.

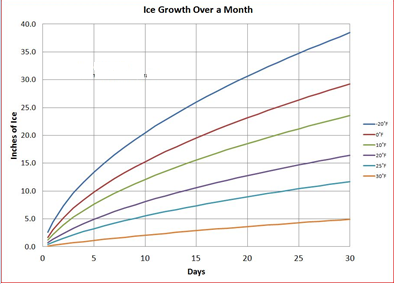
Your task is to analyze the following series of graphs, images, and temperature data and to write a convincing argument for when it is safe to drive a car onto the ice. Use the following assumptions:

* Pretend that it is December 1, 2014
* Assume that there is already 2 inches of ice on the lake
* Assume that the lake under study is Trout Lake in Coleraine, MN

**Part 3 – continues…**







Average Daily Temperature in Coleraine, MN in December 2014



Part 3 – continues…

Show all your work and your final conclusion here. Attach additional pages if necessary but be sure to submit only your best work that clearly communicates your conclusion. In addition, clearly state any additional assumptions you are making or any limitations to the work you are presenting.

Answers will vary. Look for evidence of sound mathematical thinking and accurate conclusions. Here is a sample, brief, response:

At a rate of one inch per fifteen FDD, the table shows the number of FDDs and the corresponding number of additional inches for each day in December.

|  |  |  |  |
| --- | --- | --- | --- |
| **December** | **FDDs** | **Inches of Ice** | **Total Ice Thickness** |
| 1 | 27 | 27/15 = 1.8 | 2 + 1.8 = 3.8 |
| 2 | 12 | 12/15 = 0.8 | 3.8 + 0.8 = 4.6 |
| 3 | 12 | 12/15 = 0.8 | 4.6 + 0.8 = 5.4 |
| 4 | 0 | 0 | 5.4 |
| 5 | -3 | -3/15 = -0.2 | 5.4 – 0.2 = 5.2 |
| 6 | 5 | 5/15 = 0.3 | 5.2 + 0.3 = 5.5 |
| 7 | 0 | 0 | 5.5 |
| 8 | -1 | -1/15 = -0.07 | 5.5 – 0.07 = 5.43 |
| 9 | 9 | 9/15 = 0.6 | 5.43 + 0.6 = 6.03 |
| 10 | 7 | 7/15 = 0.5 | 6.03 + 0.5 = 6.53 |
| 11 | 3 | 3/15 = 0.2 | 6.53 + 0.2 = 6.73 |
| 12 | -1 | -1/15 = -0.07 | 6.73 – 0.07 = 6.66 |
| 13 | -9 | -9/15 = -0.6 | 6.66 – 0.6 = 6.06 |
| 14 | -12 | -12/15 = -0.8 | 6.06 – 0.8 = 5.26 |
| 15 | -8 | -8/15 = -0.53 | 5.26 – 0.53 = 4.73 |
| 16 | 10 | 10/15 = 0.7 | 4.73 + 0.7 = 5.43 |
| 17 | 10 | 10/15 = 0.7 | 5.43 + 0.7 = 6.13 |
| 18 | 9 | 9/15 = 0.6 | 6.13 + 0.6 = 6.73 |
| 19 | 7 | 7/15 = 0.5 | 6.73 + 0.5 = 7.23 |
| 20 | 2 | 2/15 = 0.13 | 7.23 + 0.13 = 7.36 |
| 21 | -2 | -2/15 = -0.13 | 7.36 – 0.13 = 7.23 |
| 22 | -3 | -3/15 = -0.2 | 7.23 – 0.2 = 7.03 |
| 23 | -3 | -3/15 = -0.2 | 7.03 – 0.2 = 6.83 |
| 24 | -2 | -2/15 = -0.13 | 6.83 – 0.13 = 6.7 |
| 25 | 0 | 0 | 6.7 |
| 26 | 3 | 3/15 = 0.2 | 6.7 + 0.2 = 6.9 |
| 27 | 10 | 10/15 = 0.7 | 6.9 + 0.7 = 7.6 |
| 28 | 14 | 14/15 = 0.93 | 7.6 + 0.93 = 8.53 |
| 29 | 29 | 29/15 = 1.93 | 8.53 + 1.93 = 10.46 |
| 30 | 33 | 33/15 = 2.2 | 10.46 + 2.2 = 12.66 |
| 31 | 17 | 17/15 = 1.13 | 12.66 + 1.13 = 13.79 |

Based on the following assumptions, it is safe to drive a car on the ice from December 28 and on.

Assumptions:

* FDDs are negative at the same rate for temperatures above 32 degrees
* Rate of ice growth is the same regardless of the ice thickness (which is not really true)
* Using the average temperatures provided as the constant temperature for that 24 hour period

Part 4 – Iron Ore Mining in Minnesota

In October 2014, The Minntac Mining Operations of United States Steel passed a major milestone when it mined its two-billionth ton of material. Minntac (blending Minnesota Taconite) takes the iron ore from the ground using the method of mining known as open pit mining. The raw iron ore is made into taconite pellets and shipped to steel mills. Taconite pellets are the main source of raw material in the steel making process.

In an October 2014 press release, U.S. Steel reported that “Minntac became the nation's first iron ore mining operation to reach this production landmark (2 billion tons)

Hull Rust Mine, Hibbing, MN

when one of the mine's 35-cubic-yard shovels loaded rock

onto a 240-ton production truck at 2:06 a.m., Saturday,

October 6. The load was then taken to the Minntac Coarse

Crusher where it was ground into gravel-sized chunks to be

used in the taconite process.”

Taconite pellets

1. How many 35-cubic-yard shovel loads of ore are needed

to create 2 billion tons of ore?



2. According to the press release, “We mined our two-billionth ton 50 years after U. S. Steel broke ground for the Pilotac mine, the forerunner of Minntac.” How many shovel loads, on average, were removed *each day* over the indicated 50 years?



3. How many truck loads (at 240 ton payload per truck load) of ore are needed to haul 2 billion tons of ore?



4. Suppose (hypothetically) that the number of trucks found in #3 above were lined up, bumper to bumper. How many miles long would the line of trucks be?



Part 4 – Additional background information from the Minnesota Department of Natural Resources

www.dnr.state.mn.us/education/geology/digging/taconite.html

# Taconite

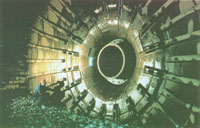
**Taconite is a low-grade iron ore.** When the high-grade natural iron ore was plentiful, taconite was considered a waste rock and not used. But as the supply of high-grade natural ore decreased, industry began to view taconite as a resource. Dr. E.W. Davis of the University of Minnesota, along with other scientists and engineers, conducted years of laboratory tests and experiments to find a way to take the iron ore out of the taconite rock. After many years of hard work, a process was developed to create taconite pellets. Taconite saved Minnesota's iron ore mining industry.

|  |  |
| --- | --- |
| **The Hull Rust Mahoning Mine in Hibbing, Minnesota**   * World's largest open pit iron ore mine * First ore shipments in 1895 (still being mined today, 114 years later) * Originally 30 separate mines * Total area: 1,591 acres * Total length: 3 1/2 miles * Greatest width: 1 1/2 miles * Greatest depth: 535 feet * Total ore shipped: About 1 billion tons * Total rock removed: About 2 billion tons (that's 4 trillion pounds!) | Hull Rust Mahoning Mine, Hibbing, MN. |

Let's look at how the taconite pellet process works.

**1. Blasting**  
Taconite is a very hard rock. Using explosives, the taconite is blasted into small pieces.

**2. Transportation**  
The taconite pieces are scooped up by electric shovels. Each shovel can hold up to 85 tons of rock! The shovels place the taconite into giant dump trucks. These trucks are as big as a house and hold up to 240 tons of taconite. The trucks take the taconite directly to the processing plant, if it is nearby, or to train cars if it is far away.

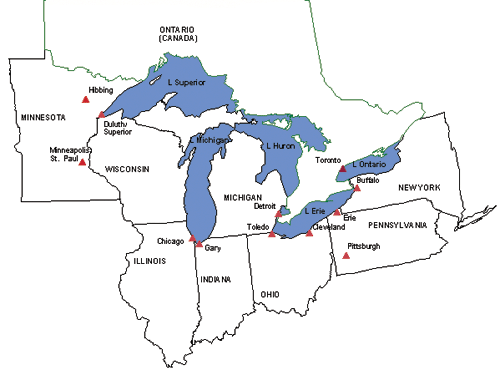


**3. Crushing**  
At the processing plant, the taconite is crushed into very small pieces by rock crushing machines. The crushers keep crushing the rock until it is the size of a marble. The rock is mixed with water and ground in rotating mills until it is as fine as powder.

**4. Separation**  
The iron ore is separated from the taconite using magnetism. The remaining rock is waste material and is dumped into tailings basins. The taconite powder with the iron in it is called concentrate.

**5. Pellets**  
The concentrate (the wet taconite powder) is rolled with clay inside large rotating cylinders. The cylinders cause the powder to roll into marble-sized balls. (This is like rolling wet, sticky snow into balls to make a snowman). The balls are then dried and heated until they are white hot. The balls become hard as they cool. The finished product is taconite pellets.

**6. Steel**  
The taconite pellets are loaded into ore ships. These ships sail on the Great Lakes to Gary, Indiana, Cleveland, Ohio and other steel-making towns. The taconite pellets are brought to the steel mills to be melted down into steel.



Taconite is mined from the Mesabi Iron Range, near Hibbing, MN. Then it is processed into pellets and moved by train--or on ore boats from Duluth--to ports and steel mills around the Great Lakes region.

Taconite process photos courtesy American Iron Ore Association and Hibbing Taconite.

s7d2.scene7.com/is/content/Caterpillar/C585483

